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The Muon Anomalous Magnetic Moment: A Harbinger For "New Physics"

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Abstract

QED, Hadronic, and Electroweak Standard Model contributions to the muon anomalous magnetic moment, $a_\mu \equiv (g_\mu - 2)/2$, and their theoretical uncertainties are scrutinized. The status and implications of the recently reported 2.6 sigma experiment vs. theory deviation $a_\mu^{\rm exp} - a_\mu^{\rm SM} = 426(165) \times 10^{-11}$ are discussed. Possible explanations due to supersymmetric loop effects with $m_{\rm SUSY} \simeq 55\sqrt{\tan\beta}$ GeV, radiative mass mechanisms at the 1–2 TeV scale and other "New Physics" scenarios are examined.

1 Introduction

Leptonic anomalous magnetic moments have traditionally provided precision tests of the Standard Model (SM) and stringent constraints on potential "New Physics" effects. In the case of the electron, comparing the extraordinary measurements of $a_e \equiv (g_e - 2)/2$ at the University of Washington [1]

$$\begin{aligned} a_{e^-}^{\exp} &= 0.001\,159\,652\,188\,4(43), \\ a_{e^+}^{\exp} &= 0.001\,159\,652\,187\,9(43), \end{aligned} \tag{1}$$

with the prediction [2, 3, 4, 5]

$$\begin{aligned} a_e^{\mathrm{SM}} &= \frac{\alpha}{2\pi} - 0.328\,478\,444\,00 \left(\frac{\alpha}{\pi}\right)^2 + 1.181\,234\,017 \left(\frac{\alpha}{\pi}\right)^3 \\ &\quad -1.5098(384) \left(\frac{\alpha}{\pi}\right)^4 + 1.66(3) \times 10^{-12} (\text{hadronic \& electroweak loops}) \end{aligned} \tag{2}$$

currently provides the best determination of the fine structure constant [6],

$$\alpha^{-1}(a_e) = 137.035\,999\,58(52). \tag{3}$$

To test the Standard Model requires comparison with an alternative measurement of α with comparable accuracy. Unfortunately, the next best determination of α, from the quantum Hall effect [2],

$$\alpha^{-1}(qH) = 137.036\,003\,00(270), \tag{4}$$

has a considerably larger error. If one assumes that $\left|\Delta a_e^{\text{New Physics}}\right| \simeq m_e^2/\Lambda^2$, where Λ approximates the scale of "New Physics", then the agreement between $\alpha^{-1}(a_e)$ and $\alpha^{-1}(qH)$ currently probes $\Lambda \lesssim \mathcal{O}\,(100 \text{ GeV})$. To access the much more interesting $\Lambda \sim \mathcal{O}\,(\text{TeV})$ region would require an order of magnitude improvement in $a_e^{\exp}$ (technically feasible [7]), an improved calculation of the 4-loop QED contribution to a_e^{SM} and a much better independent measurement of α^{-1} by almost two orders of magnitude. The last requirement, although extremely challenging, is perhaps most likely to come [6] from combining the already precisely measured Rydberg constant with a much better determination of m_e.

We should note that for potential "New Physics" (NP) effects linear in the electron mass, $\Delta a_e^{\mathrm{NP}} \sim m_e/\Lambda$, naively, one is currently probing a much more impressive $\Lambda \sim \mathcal{O}\,(10^7 \text{ GeV})$ and the possible advances described above would explore $\mathcal{O}\,(10^9 \text{ GeV})$! However, we subsequently argue that such linear "New Physics" effects are generally

misleading because the associated physics is likely to also give unacceptably large corrections to the electron mass.

Improvements in the measurement of the muon's anomalous magnetic moment have also been impressive. A series of dedicated experiments at CERN that ended in 1977 found [8]

$$a_\mu^{\rm exp} = 116\,592\,300(840) \times 10^{-11} \qquad \text{(CERN 1977)}. \tag{5}$$

More recently, an ongoing experiment (E821) at Brookhaven National Laboratory has been running with much higher statistics and a very stable, well measured magnetic field in its storage ring. Based on μ^+ data taken through 1998, combined with the earlier CERN result in (5), it reported [9]

$$a_\mu^{\rm exp} = 116\,592\,050(460) \times 10^{-11} \qquad \text{(CERN'77 + BNL'98)}. \tag{6}$$

That group has just announced a much higher statistics result based on 1999 data [10],

$$a_\mu^{\rm exp} = 116\,592\,020(160) \times 10^{-11} \quad \text{(BNL'99)}. \tag{7}$$

Their finding is very consistent with Eq. (6). When simply averaged together, we find

$$a_\mu^{\rm exp}(\text{Average}) = 116\,592\,023(151) \times 10^{-11} \quad \text{(CERN'77 + BNL'98 \& '99)}. \tag{8}$$

The ultimate goal of the experiment (which has its final scheduled run with μ^- during 2001) is $\pm 40 \times 10^{-11}$, about a factor of 20 improvement relative to the classic CERN experiments and a factor of 3.5 better than the average in Eq. (8). Even the inclusion of already existing data from the 2000 run is expected to reduce the error in Eq. (8) by more than a factor of 2 within the coming year.

Although $a_\mu^{\rm exp}$ is currently about 350 times less precise than $a_e^{\rm exp}$, it is much more sensitive to hadronic and electroweak quantum loops as well as "New Physics" effects, since such contributions [11] are generally proportional to m_l^2. The $m_\mu^2/m_e^2 \simeq 40\,000$ enhancement more than compensates for the reduced experimental precision and makes $a_\mu^{\rm exp}$ a much better probe of short-distance phenomena. Indeed, as we later illustrate, a deviation in $a_\mu^{\rm exp}$ from the Standard Model prediction, $a_\mu^{\rm SM}$, even at its current level of sensitivity can quite naturally be interpreted as the appearance of "New Physics" such as supersymmetry at 100-450 GeV, or other even higher scale phenomena, exciting prospects. Of course, before making such an interpretation, one must have a reliable theoretical prediction for $a_\mu^{\rm SM}$ with which to compare, an issue that we address in the next section.

Before leaving the comparison between a_e^{exp} and a_μ^{exp}, we should remark that for cases where "New Physics" contributions to a_l scale as m_l/Λ, roughly equal sensitivity in Λ ($\sim 10^7$ GeV) currently exists for both types of measurements. However, as previously mentioned, such effects are in our view artificial.

2 Standard Model Prediction For a_μ

2.1 QED Contribution

The QED contribution to a_μ has been computed (or estimated) through 5 loops [5, 2]

$$\begin{aligned} a_\mu^{\mathrm{QED}} &= \frac{\alpha}{2\pi} + 0.765\,857\,376(27)\left(\frac{\alpha}{\pi}\right)^2 + 24.050\,508\,98(44)\left(\frac{\alpha}{\pi}\right)^3 \\ &\quad + 126.07(41)\left(\frac{\alpha}{\pi}\right)^4 + 930(170)\left(\frac{\alpha}{\pi}\right)^5. \end{aligned} \tag{9}$$

Growing coefficients in the α/π expansion reflect the presence of large $\ln\frac{m_\mu}{m_e} \simeq 5.3$ terms coming from electron loops. Employing the value of α from a_e in eq. (3) leads to

$$a_\mu^{\mathrm{QED}} = 116\,584\,705.7(2.9) \times 10^{-11}. \tag{10}$$

The current uncertainty is well below the $\pm 40 \times 10^{-11}$ ultimate experimental error anticipated from E821 and should, therefore, play no essential role in the confrontation between theory and experiment.

2.2 Hadronic Loop Corrections

Starting at $\mathcal{O}(\alpha^2)$, hadronic loop effects contribute to a_μ via vacuum polarization (see Fig. 1). A first principles QCD calculation of that effect does not exist. Fortunately, it is possible to evaluate the leading effect via the dispersion integral [12]

$$a_\mu^{\mathrm{Had}}(\text{vac. pol.}) = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} \mathrm{d}s\, K(s)\ \sigma^0(s)_{e^+e^-\to\mathrm{hadrons}}, \tag{11}$$

where $\sigma^0(s)_{e^+e^-\to\mathrm{hadrons}}$ means QED vacuum polarization and some other extraneous radiative corrections (e.g. initial state radiation) have been subtracted from

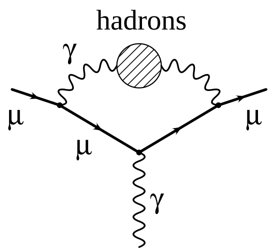


Figure 1: Leading hadronic vacuum polarization corrections to a_μ.

measured $e^+e^- \to$ hadrons cross sections, and

$$
\begin{aligned}
K(s) &= x^2\left(1-\frac{x^2}{2}\right)+(1+x)^2\left(1+\frac{1}{x^2}\right)\left[\ln(1+x)-x+\frac{x^2}{2}\right]+\frac{1+x}{1-x}\,x^2\ln x \\
x &= \frac{1-\sqrt{1-4m_\mu^2/s}}{1+\sqrt{1-4m_\mu^2/s}}.
\end{aligned}
\tag{12}
$$

Detailed studies of eq. (11) have been carried out by a number of authors [13, 14, 15, 16, 17, 18, 19, 20]. The most precise published analysis to date, due to Davier and Höcker [14, 15, 16], found

$$a_\mu^{\rm Had}(\text{vac. pol.}) = 6924(62)\times 10^{-11}. \tag{13}$$

It employed experimental e^+e^- data, hadronic tau decays, perturbative QCD and sum rules to minimize the uncertainty in that result. The contributions coming from various energy regions are illustrated in Table 1.

It is clear from Table 1 that the final result and its uncertainty are dominated by the low energy region. In fact, the $\rho(770\text{ MeV})$ resonance provides about 72% of the total hadronic contribution to $a_\mu^{\rm Had}$(vac. pol.).

To reduce the uncertainty in the ρ resonance region, Davier and Höcker employed $\Gamma(\tau\to\nu_\tau\pi^-\pi^0)/\Gamma(\tau\to\nu_\tau\bar{\nu}_e e^-)$ data to supplement $e^+e^-\to\pi^+\pi^-$ cross-sections. In the $I=1$ channel they are related by isospin. Currently, tau decay data is experimentally more precise and in principle has the advantage of being self-normalizing if both $\tau\to\nu_\tau\pi^-\pi^0$ and $\tau\to\nu_\tau\overline{\nu}_e e$ are both measured in the same experiment.

An issue in the use of tau decay data is the magnitude of isospin violating corrections due to QED and the m_d-m_u mass difference. A short-distance QED correction [21] of about -2% was applied to the hadronic tau decay data and isospin violating effects such as $m_{\pi^\pm}-m_{\pi^0}$ phase space and $\rho^\pm-\rho^0$ differences have been accounted

Table 1: Contributions to $a_\mu^{\rm Had}$(vac. pol.) from different energy regions as found by Davier and Höcker [14, 15, 16].

$\sqrt{s}$ (GeV)	$a_\mu^{\rm Had}$(vac. pol.) $\times 10^{11}$
$2m_\pi - 1.8$	6343 ± 60
$1.8 - 3.7$	338.7 ± 4.6
$3.7 - 5 + \psi(1S, 2S)$	143.1 ± 5.4
$5 - 9.3$	68.7 ± 1.1
$9.3 - 12$	12.1 ± 0.5
$12 - \infty$	18.0 ± 0.1
Total	6924 ± 62

for. Other uncorrected differences are estimated to be about $\pm 0.5\%$ and included in the hadronic uncertainty.

Although the $\pm 0.5\%$ error assigned to the use of tau decay data appears reasonable, it has been questioned [22, 23]. More recent preliminary $e^+e^- \to \pi^+\pi^-$ data from Novosibirsk [22] seems to suggest a potential difference with corrected hadronic tau decays which could compromise the estimated $a_\mu^{\rm Had}$ in Eq. (13). It is not clear at this time whether the difference is due to additional isospin violating corrections to hadronic tau decays, normalization issues [24], or radiative corrections to $e^+e^- \to$ hadrons data which must be accounted for in any precise comparison [25]. Resolution of this issue is extremely important.

A more conservative approach might be to ignore the tau data and use QCD theory input as little as possible. In an (unpublished) update of earlier work [17], Jegerlehner found from such an approach

$$a_\mu^{\rm Had}(\text{vac. pol.}) = 6988(111) \times 10^{-11} \quad \text{(Jegerlehner 2000, preliminary [26]).} \quad (14)$$

Within their quoted errors, Eqs. (13) and (14) agree but the central values differ by 64×10^{-11}. The sign of the difference between Eq. (14) and Eq. (13) may be a little misleading, since tau data tends to favor a larger contribution to $a_\mu^{\rm Had}$ from the ρ than $e^+e^- \to$ hadrons [24]. Anticipated new results for $e^+e^- \to$ hadrons at Novosibirsk should reduce the uncertainty in (14) by nearly a factor of 2 without requiring tau data. It will be interesting to see what happens to its central value.

Evaluation of the 3-loop hadronic vacuum polarization contribution to a_μ has been

updated to [27, 18]

$$\Delta a_\mu^{\rm Had}(\text{vac. pol.}) = -100(6) \times 10^{-11}. \tag{15}$$

Light-by-light hadronic diagrams have been evaluated using chiral perturbation theory. An average [14, 15, 16] of two recent studies [28, 29] gives

$$\Delta a_\mu^{\rm Had}(\text{light-by-light}) = -85(25) \times 10^{-11}. \tag{16}$$

Adding those contributions to Eqs. (13) leads to the total hadronic contribution

$$a_\mu^{\rm Had} = 6739(67) \times 10^{-11} \quad \text{(Davier \& Höcker, 1998)} \tag{17}$$

which we will subsequently use in comparison of theory and experiment. However, we note that a more conservative approach might employ a larger uncertainty such as found using Jegerlehner's unpublished result in Eq. (14),

$$a_\mu^{\rm Had} = 6803(114) \times 10^{-11} \quad \text{(Jegerlehner 2000, unpublished).} \tag{18}$$

At the very least, one should be mindful of the difference between the two and the need to further justify the use of tau decay data and low-energy perturbative QCD. The uncertainties in those results represent the main theoretical error in $a_\mu^{\rm SM}$. It would be very valuable to supplement the above evaluation of $a_\mu^{\rm Had}$ with lattice calculations (for the light-by-light contribution) and further improved e^+e^- data (beyond ongoing experiments). An ultimate goal of $\pm 40 \times 10^{-11}$ or smaller appears to be within reach and is well matched to the prospectus of experiment E821 at Brookhaven which aims for a similar level of accuracy.

2.3 Electroweak corrections

The one-loop electroweak radiative corrections to a_μ (see Fig. 2) are predicted in the Standard Model to be [30, 31, 32, 33, 34, 35, 36]

$$\begin{aligned} a_\mu^{\rm EW}(1\,\text{loop}) &= \frac{5}{3}\frac{\mathrm{G}_\mu \mathrm{m}_\mu^2}{8\sqrt{2}\pi^2} \\ &\times \left[1 + \frac{1}{5}(1-4\sin^2\theta_W)^2 + \mathcal{O}\left(\frac{m_\mu^2}{M^2}\right)\right] \\ &\approx 195 \times 10^{-11} \end{aligned} \tag{19}$$

where $G_\mu = 1.16637(1)\times 10^{-5}\ \text{GeV}^{-2}$, $\sin^2\theta_W \equiv 1 - M_W^2/M_Z^2 \simeq 0.223.$ and $M = M_W$ or $M_{\rm Higgs}$. The original goal of E821 at Brookhaven was to measure that predicted

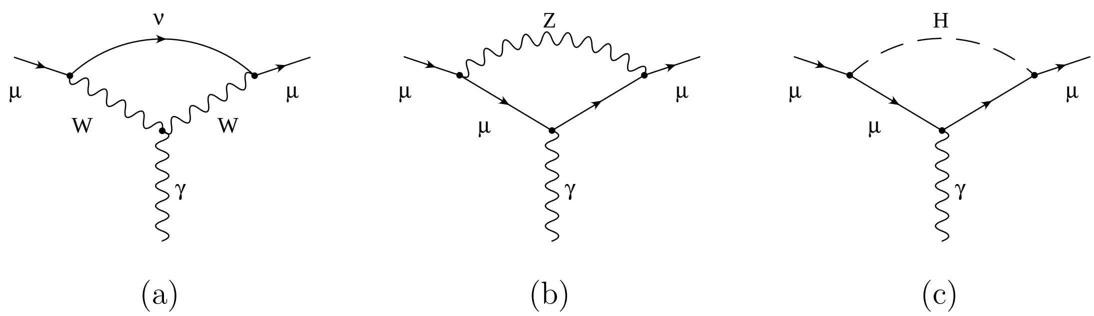


Figure 2: One-loop electroweak radiative corrections to a_μ.

effect at about the 5 sigma level (assuming further reduction in the hadronic uncertainty). Subsequently, it was pointed out [37] that two-loop electroweak contributions are relatively large due to the presence of $\ln m_Z^2/m_\mu^2 \simeq 13.5$ terms. A full two-loop calculation [38, 39], including low-energy hadronic electroweak loops [40, 39], found for $m_H \simeq 150$ GeV (with little sensitivity to the exact value)

$$a_\mu^{\mathrm{EW}}(2\text{ loop}) = -43(4) \times 10^{-11}, \tag{20}$$

where the quoted error is a conservative estimate of hadronic, Higgs, and higher-order corrections. Combining eqs. (19) and (20) gives the electroweak contribution

$$a_\mu^{\mathrm{EW}} = 152(4) \times 10^{-11}. \tag{21}$$

Higher-order leading logs of the form $(\alpha \ln m_Z^2/m_\mu^2)^n$, $n = 2, 3, \ldots$ can be computed via renormalization group techniques [41]. Due to cancellations between the running of α and anomalous dimension effects, they give a relatively small $+0.5 \times 10^{-11}$ contribution to a_μ^{EW}. It is safely included in the uncertainty of eq. (21).

2.4 Comparison with Experiment

The complete Standard Model prediction for a_μ is

$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{Had}} + a_\mu^{\mathrm{EW}}. \tag{22}$$

Combining eqs. (10), (17) and (21), one finds

$$a_\mu^{\mathrm{SM}} = 116\,591\,597(67) \times 10^{-11}, \tag{23}$$

or, using eqs. (10), along with the more conservative (18) and (21), $a_\mu^{\rm SM} = 116\,591\,661(114)\times 10^{-11}$. Comparing Eq. (23) with the current experimental average in Eq. (8) gives

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 426 \pm 165 \times 10^{-11}. \qquad (24)$$

The roughly 2.6σ difference is very exciting. It may be an indicator or harbinger of contributions from "New Physics" beyond the Standard Model. At 90% CL, one finds

$$215 \times 10^{-11} \leq a_\mu(\text{New Physics}) \leq 637 \times 10^{-11}, \qquad (25)$$

which suggests a relatively large "New Physics" effect, even larger than the predicted electroweak contribution, is starting to be seen. As we show in the next section, several realistic examples of "New Physics" could quite easily lead to $a_\mu(\text{New Physics}) \sim \mathcal{O}\left(426 \times 10^{-11}\right)$ and might be responsible for the apparent deviation. If that is the case, the difference in Eq. (24) should increase to a 6 or more sigma effect as E821 is completed and the hadronic uncertainties in $a_\mu^{\rm SM}$ are further reduced.

3 "New Physics" effects

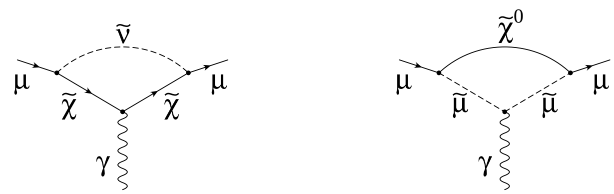
Since the anomalous magnetic moment comes from a dimension 5 operator, "New Physics" (i.e. beyond the Standard Model expectations) will contribute to a_μ via induced quantum loop effects (rather than tree level). Whenever a new model or Standard Model extension is proposed, such effects are examined and $a_\mu^{\rm exp} - a_\mu^{\rm SM}$ is often employed to constrain or rule it out.

In this section we describe several examples of interesting "New Physics" probed by $a_\mu^{\rm exp} - a_\mu^{\rm SM}$. Rather than attempting to be inclusive, we concentrate on two general scenarios: 1) Supersymmetric loop effects which can be substantial and would be heralded as the most likely explanation if the deviation in $a_\mu^{\rm exp}$ is confirmed and 2) Models of radiative muon mass generation which predict $a_\mu(\text{New Physics}) \sim m_\mu^2/M^2$ where M is the scale of "New Physics". Either case is capable of explaining the apparent deviation in $a_\mu^{\rm exp} - a_\mu^{\rm SM}$ exhibited in Eq. (24). Other examples of potential "New Physics" contributions to a_μ are only briefly discussed.

3.1 Supersymmetry

The supersymmetric contributions to a_μ stem from smuon–neutralino and sneutrino-chargino loops (see Fig. 3). They include 2 chargino and 4 neutralino states and

could in principle entail slepton mixing and phases. Depending on SUSY masses, mixing and other parameters, the contribution of $a_\mu^{\rm SUSY}$ can span a broad range of possibilities. Studies have been carried out for a variety of models where the parameters are specified. Here we give a generic discussion primarily intended to illustrate the strong likelihood that evidence for supersymmetry can be inferred from $a_\mu^{\rm exp}$ and may in fact be the natural explanation for the apparent deviation from SM theory reported by E821.



(a) (b)

Figure 3: Supersymmetric loops contributing to the muon anomalous magnetic moment.

Early studies of the supersymmetric contributions $a_\mu^{\rm SUSY}$ were carried out in the context of the minimal SUSY standard model (MSSM) [42, 43, 44, 45, 46, 47, 48, 49], in an E_6 string-inspired model [50, 51], and in an extension of the MSSM with an additional singlet [52, 53]. An important observation was made in [54], namely that some of the contributions are enhanced by the ratio of Higgs' vacuum expectation values, $\tan\beta \equiv \langle\Phi_2\rangle/\langle\Phi_1\rangle$, which in some models is large (in some cases of order $m_t/m_b \approx 40$). In addition, larger values of $\tan\beta \gtrsim 2$ are generally in better accord with the recent LEP II Higgs mass bound $m_H \gtrsim 113$ GeV and, therefore, currently favored. The main contribution is generally due to the chargino-sneutrino diagram (Fig. 3a), which is enhanced by a Yukawa coupling in the muon-sneutrino-Higgsino vertex (charginos are admixtures of Winos and Higgsinos).

The leading effect from Fig. 3a is approximately given in the large $\tan\beta$ limit by

$$\left|a_\mu^{\rm SUSY}\right| \simeq \frac{\alpha(M_Z)}{8\pi\sin^2\theta_W}\,\frac{m_\mu^2}{\widetilde{m}^2}\,\tan\beta\left(1-\frac{4\alpha}{\pi}\ln\frac{\widetilde{m}}{m_\mu}\right), \tag{26}$$

where $\widetilde{m} = m_{\rm SUSY}$ represents a typical SUSY loop mass. (Chargino- and sneutrino-masses are actually assumed degenerate in that expression [55]; otherwise, $\widetilde{m}$ is approximately the heavier mass scale.) Also, we have included a 7–8% suppression factor due to leading 2-loop EW effects. Like most "New Physics" effects, SUSY

loops contribute directly to the dimension 5 magnetic dipole operator. In that case, they are subject to the same EW suppression factor as the W loop contribution to $a_\mu^{\rm EW}$. From the calculation in Ref. [38, 41], one finds a leading log suppression factor

$$1 - \frac{4\alpha}{\pi} \ln \frac{M}{m_\mu} \tag{27}$$

where M is the characteristic "New Physics" scale. For $M \sim 200$ GeV, that factor corresponds to about a 7% reduction.

Numerically, one expects in the large $\tan\beta$ regime (after a small negative contribution from Fig. 3b is included, again assuming degenerate masses)

$$\left|a_\mu^{\rm SUSY}\right| \simeq 130 \times 10^{-11} \left(\frac{100 \text{ GeV}}{\widetilde{m}}\right)^2 \tan\beta, \tag{28}$$

where $a_\mu^{\rm SUSY}$ generally has the same sign as the μ-parameter in SUSY models.

Ref. [54] found that E821 will be a stringent test of a class of supergravity models. However, in the minimal SU(5) SUGRA model, $\tan\beta$ is already severely constrained by the proton decay lifetime and no significant $a_\mu^{\rm SUSY}$ is possible. Extended models, notably SU(5)$\times$U(1) escape that bound and can induce large effects.

Supersymmetric effects in a_μ were subsequently computed in a variety of models. Constraints on MSSM were examined in [55, 56]. MSSM with large CP-violating phases was studied in [57]. Ref. [58] examined models with a superlight gravitino. Detailed studies of $a_\mu^{\rm SUSY}$ were carried out in models constrained by various assumptions on the SUSY-breaking mechanism: gauge-mediated [59, 60], SUGRA [61, 62, 63], and anomaly-mediated [64].

Rather than focusing on a specific model, we simply employ for illustration the large $\tan\beta$ approximate formula in eq. (28) with degenerate SUSY masses and the current constraint in eq. (24). Then we find (for positive sgn(μ)) from comparison with Eq. (24)

$$\tan\beta \left(\frac{100 \text{ GeV}}{\widetilde{m}}\right)^2 \simeq 3.3 \pm 1.3, \tag{29}$$

or

$$\widetilde{m} \simeq (55 \text{ GeV})\sqrt{\tan\beta}. \tag{30}$$

(Of course, in specific models with non-degenerate gauginos and sleptons, a more detailed analysis is required, but here we only want to illustrate roughly the scale

of supersymmetry being probed.) Negative μ models give the opposite sign contribution to a_μ and are strongly disfavored.

For large $\tan\beta$ in the range $4 \sim 40$, where the approximate results given above should be valid, one finds (assuming $\widetilde{m} > 100$ GeV from other experimental constraints)

$$\widetilde{m} \simeq 100 - 450 \text{ GeV} \tag{31}$$

precisely the range where SUSY particles are often expected. If supersymmetry in the mass range of Eq. (31) with relatively large $\tan\beta$ is responsible for the apparent $a_\mu^{\rm exp} - a_\mu^{\rm SM}$ difference, it will have many dramatic consequences. Besides expanding the known symmetries of Nature and our fundamental notion of space-time, it will impact other new exploratory experiments. Indeed, for $\widetilde{m} \simeq 100 - 450$ GeV, one can expect a plethora of new SUSY particles to be discovered soon, either at the Fermilab 2 TeV $p\bar{p}$ collider or certainly at the LHC 14 TeV pp collider which is expected to start running 2006.

Large $\tan\beta$ supersymmetry can also have other interesting loop–induced low energy consequences beyond a_μ. For example, it can affect $b \to s\gamma$. Even for the muon, “New Physics” in a_μ is likely to suggest potentially observable $\mu \to e\gamma$, $\mu^- N \to e^- N$ and a muon electric dipole moment (edm), depending on the degree of flavor mixing and CP violating phases. Searches for these phenomena are now entering an exciting phase, with a new generation of experiments being proposed or constructed. The decay $\mu \to e\gamma$ will be searched for with 2×10^{-14} single event sensitivity (SES) at the Paul Scherrer Institute [65]. The MECO experiment at BNL [66] will search for the muon-electron conversion, $\mu^-\text{Al} \to e^-\text{Al}$, with 2×10^{-17} SES. A proposal has been made [67] to search for the muon’s electric dipole moment with sensitivity of about 10^{-24} e·cm with the BNL muon storage ring. Certainly, the hint of supersymmetry suggested by $a_\mu^{\rm exp}$ will provide strong additional motivation to extend such studies both theoretically and experimentally.

3.2 Radiative Muon Mass Models

The relatively light masses of the muon and most other known fundamental fermions could suggest that they are radiatively loop induced by “New Physics” beyond the Standard Model. Although no compelling model exists, the concept is very attractive as a natural scenario for explaining the flavor mass hierarchy, i.e. why most fermion masses are so much smaller than the electroweak scale ~ 250 GeV.

The basic idea is to start off with a naturally zero bare fermion mass due to an underlying chiral symmetry. The symmetry is broken in the fermion 2-point function

by quantum loop effects. They lead to a finite calculable mass which depends on the mass scales, coupling strengths and dynamics of the underlying symmetry breaking mechanism. In such a scenario, one generically expects for the muon

$$m_\mu \propto \frac{g^2}{16\pi^2} M_F, \tag{32}$$

where g is some new interaction coupling strength and $M_F \sim 100 - 1000$ GeV is a heavy scale associated with chiral symmetry breaking and perhaps electroweak symmetry breaking. Of course, there may be other suppression factors at work in Eq. (32) that keep the muon mass small.

Whatever source of chiral symmetry breaking is responsible for generating the muon's mass will also give rise to non-Standard Model contributions in a_μ. Indeed, fermion masses and anomalous magnetic moments are intimately connected chiral symmetry breaking operators. Remarkably, in such radiative scenarios, the additional contribution to a_μ is quite generally given by [68, 69]

$$a_\mu(\text{New Physics}) \simeq C \frac{m_\mu^2}{M^2}, \qquad C \simeq \mathcal{O}\left(1\right), \tag{33}$$

where M is some physical high mass scale associated with the "New Physics" and C is a model-dependent number roughly of order 1 (it can even be larger). M need not be the same scale as M_F in eq. (32). In fact, M is usually a somewhat larger gauge or scalar boson mass responsible for mediating the chiral symmetry breaking interaction. The result in eq. (33) is remarkably simple in that it is largely independent of coupling strengths, dynamics, etc. Furthermore, rather than exhibiting the usual $g^2/16\pi^2$ loop suppression factor, a_μ(New Physics) is related to m_μ^2/M^2 by a (model dependent) constant, C, roughly of $\mathcal{O}\left(1\right)$.

To demonstrate how the relationship in eq. (33) arises, we first consider a simple toy model example [69] for muon mass generation which is graphically depicted in Fig. 4.

If the muon is massless in lowest order (i.e. no bare m_μ^0 is possible due to a symmetry), but couples to a heavy fermion F via scalar, S, and pseudoscalar, P, bosons with couplings g and $g\gamma_5$ respectively, then the diagrams give rise to

$$\begin{aligned} m_\mu &\simeq \frac{g^2}{16\pi^2} M_F \left(\frac{M_S^2}{M_S^2 - M_F^2} \ln \frac{M_S^2}{M_F^2} - \frac{M_P^2}{M_P^2 - M_F^2} \ln \frac{M_P^2}{M_F^2} \right) && (34) \\ &\to \frac{g^2}{16\pi^2} M_F \ln \left(\frac{M_S^2}{M_P^2} \right) \quad (M_{S,P} \gg M_F). && (35) \end{aligned}$$

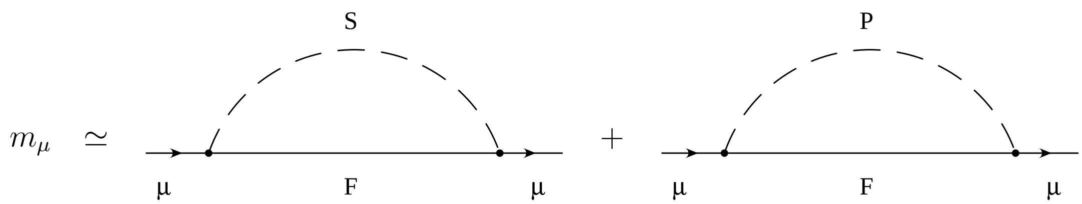


Figure 4: Example of a pair of one-loop diagrams which can induce a finite radiative muon mass.

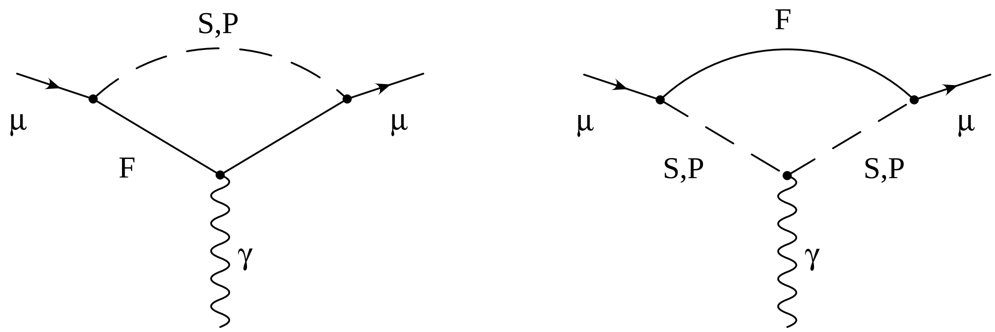


Figure 5: Potential diagrams that can contribute to the anomalous magnetic moment in radiative muon mass models.

Note that short-distance ultraviolet divergences have canceled and the induced mass vanishes in the chirally symmetric limit $M_S = M_P$. If we attach a photon to the heavy internal fermion, F, or boson S or P (assumed to carry fractions Q_F and $1 - Q_F$ of the muon charge, respectively), then a new contribution to a_μ is also induced (see Fig. 5). One finds

$$\begin{aligned} a_\mu(\text{New Physics}) \quad = \quad & \frac{g^2}{16\pi^2}\{Q_F\,[f_N(M_P) - f_N(M_S)] \\ & +(1-Q_F)\,[f_C(M_P) - f_C(M_S)]\}\,, \end{aligned} \tag{36}$$

with

$$f_N(M_X) = \frac{m_\mu M_F}{(M_X^2 - M_F^2)^3}\left(3M_X^4 + M_F^4 - 4M_X^2 M_F^2 - 2M_X^4 \ln\frac{M_X^2}{M_F^2}\right), \tag{37}$$

$$f_C(M_X) = \frac{m_\mu M_F}{(M_X^2 - M_F^2)^3}\left(M_X^4 - M_F^4 - 2M_F^2 M_X^2 \ln\frac{M_X^2}{M_F^2}\right). \tag{38}$$

In the limit $M_{S,P} \gg M_F$ and $Q_F = 1$, one finds [69]

$$a_\mu(\text{New Physics}) \simeq \frac{g^2}{8\pi^2}\frac{m_\mu M_F}{M_P^2}\left(\frac{M_P^2}{M_S^2}\ln\frac{M_S^2}{M_F^2} - \ln\frac{M_P^2}{M_F^2}\right), \tag{39}$$

while for $Q_F = 0$

$$a_\mu(\text{New Physics}) \simeq \frac{g^2}{8\pi^2}\frac{m_\mu M_F}{M_P^2}\left(1 - \frac{M_P^2}{M_S^2}\right). \tag{40}$$

The induced a_μ(New Physics) also vanishes in the $M_S = M_P$ chiral symmetry limit. Interestingly, a_μ(New Physics) exhibits a linear rather than quadratic dependence on m_μ at this point. Recall, that in section 1 we said that such a feature was misleading or artificial. Our subsequent discussion should clarify that point.

Although eqs. (35) and (39) both depend on unknown parameters such as g and M_F, those quantities largely cancel when we combine both expressions. One finds

$$\begin{aligned}
a_\mu(\text{New Physics}) &\simeq C\frac{m_\mu^2}{M_P^2}, \\
C &= 2\left[1 - \left(1 - \frac{M_P^2}{M_S^2}\right)\ln\frac{M_S^2}{M_F^2}/\ln\frac{M_S^2}{M_P^2}\right] \quad \text{for } Q_F = 1, \\
C &= \left(1 - \frac{M_P^2}{M_S^2}\right)/\ln\frac{M_S^2}{M_P^2} \quad \text{for } Q_F = 0,
\end{aligned} \tag{41}$$

where C is very roughly $\mathcal{O}(1)$. It can actually span a broad range and take on either sign, depending on the M_S/M_P ratio and Q_F. A loop produced a_μ(New Physics) effect that started out at $\mathcal{O}(g^2/16\pi^2)$ has effectively been promoted to $\mathcal{O}(1)$ by absorbing the couplings and M_F factor into m_μ. Along the way, the linear dependence on m_μ has been replaced by a more natural quadratic dependence.

An alternative prescription for radiatively generating fermion masses involves new strong dynamics, e.g. extended technicolor. In such scenarios, technifermions acquire, via new strong dynamics, dynamical self-energies

$$\Sigma_F(p) \simeq m_F\left(\frac{\Lambda^2}{\Lambda^2 - p^2}\right)^{1-\frac{\gamma}{2}}, \tag{42}$$

where $0 < \gamma < 2$ is an anomalous dimension, $m_F \simeq \mathcal{O}(300\text{ GeV})$, and Λ is the new strong interaction scale $\sim \mathcal{O}(1\text{ TeV})$.

Ordinary fermions such as the muon receive loop induced masses via the diagram in Fig. 6.

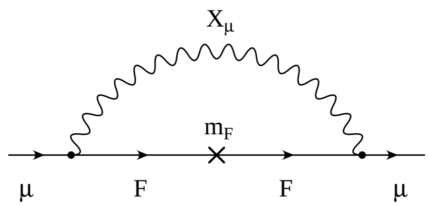


Figure 6: Extended technicolor-like diagram responsible for generating the muon mass.

The extended gauge boson X_μ links μ and F via the non-chiral coupling

$$g\gamma_\mu \left(a\frac{1-\gamma_5}{2} + b\frac{1+\gamma_5}{2} \right) \tag{43}$$

and gives rise to a mass [68, 69]

$$m_\mu \simeq \frac{g^2 ab}{4\pi^2} m_F \left(\frac{\Lambda}{m_{X_\mu}} \right)^{2-\gamma} \Gamma\left(\frac{\gamma}{2}\right) \Gamma\left(1-\frac{\gamma}{2}\right), \tag{44}$$

where $\Gamma(x)$ is the Gamma function. Notice, the ultraviolet divergence at $\gamma = 2$ which corresponds to a non-dynamical m_F.

If we attach a photon to the internal fermion line in Fig. 6 (assumed here to have charge -1), an anomalous magnetic moment contribution is induced. One finds

$$a_\mu(\text{New Dynamics}) \simeq \frac{g^2 ab}{2\pi^2} \frac{m_\mu m_F}{m_{X_\mu}^2} \left(\frac{\Lambda}{m_{X_\mu}} \right)^{2-\gamma} \frac{\Gamma\left(2-\frac{\gamma}{2}\right) \Gamma\left(\frac{\gamma}{2}\right)}{1+\frac{\gamma}{2}}. \tag{45}$$

Again we see only a linear dependence on m_μ. However, when Eq. (44) and (45) are combined, one finds

$$a_\mu(\text{New Dynamics}) \simeq 2\left(\frac{2-\gamma}{2+\gamma} \right) \frac{m_\mu^2}{m_{X_\mu}^2}, \tag{46}$$

i.e. the generic result $\mathcal{O}(1)\, m_\mu^2/M^2$ where M is the "new physics" scale (here the extended-techniboson mass) emerges.

A similar relationship, $a_\mu(\text{New Physics}) \simeq C m_\mu^2/M^2$, has been found in more realistic multi-Higgs models [70], SUSY with soft masses [71], etc. It is also a natural expectation in composite models [72, 73, 74] or some models with large extra dimensions [75, 76], although studies of such cases have not necessarily made that

same connection. Basically, the requirement that m_μ remain relatively small in the presence of new chiral symmetry breaking interactions forces a_μ(New Physics) to effectively exhibit a quadratic m_μ^2 dependence.

For models of the above variety, where $|a_\mu(\text{New Physics})| \simeq m_\mu^2/M^2$, the current constraint in eq. (25) suggests (very roughly)

$$M \simeq 1 - 2 \text{ TeV}. \tag{47}$$

Of course, for a specific model, one must check that the sign of the induced a_μ^{NP} is in accord with experiment (i.e. it should be positive).

Such a scale of "New Physics" could be quite natural in multi-Higgs radiative mass models and soft SUSY mass scenarios. It would be somewhat low for dynamical symmetry breaking, compositeness and extra dimension models, however, confirmation of an a_μ^{exp} deviation will certainly lead to all possibilities being revisited.

3.3 Other "New Physics" Examples

3.3.1 Anomalous W Boson Properties

Anomalous W boson magnetic dipole and electric quadrupole moments can also lead to a deviation in a_μ from SM expectations. We generalize the γWW coupling such that the W boson magnetic dipole moment is given by

$$\mu_W = \frac{e}{2m_W}(1 + \kappa + \lambda) \tag{48}$$

and electric quadrupole moment by

$$Q_W = -\frac{e}{2m_W}(\kappa - \lambda) \tag{49}$$

where $\kappa = 1$ and $\lambda = 0$ in the Standard Model, i.e. the gyromagnetic ratio $g_W = \kappa + 1 = 2$. For non-standard couplings, one obtains the additional one loop contribution to a_μ given by [77, 78, 79, 80, 81]

$$a_\mu(\kappa, \lambda) \simeq \frac{G_\mu m_\mu^2}{4\sqrt{2}\pi^2}\left[(\kappa - 1)\ln\frac{\Lambda^2}{m_W^2} - \frac{1}{3}\lambda\right], \tag{50}$$

where Λ is the high momentum cutoff required to give a finite result. It presumably corresponds to the onset of "New Physics" such as the W compositeness scale, or

new strong dynamics. Higher order electroweak loop effects reduce that contribution by roughly the suppression in Eq. (27), i.e. $\sim 9\%$.

For $\Lambda \simeq 1$ TeV, the deviation in Eq. (24) corresponds to

$$\kappa - 1 = 0.37 \pm 0.14. \tag{51}$$

Such a large deviation from Standard Model expectations, $\kappa = 1$, is already ruled out by $e^+e^- \to W^+W^-$ data at LEP II which gives [82, 83]

$$\kappa - 1 = 0.04 \pm 0.08 \qquad \text{(LEP II)}. \tag{52}$$

One could reduce the requirement in Eq. (51) somewhat by assuming a much larger Λ cuoff in Eq. (50). However, it is generally felt that $\kappa - 1$ and Λ should be inversely correlated. For example $\kappa - 1 \sim m_W/\Lambda$ or $(m_W/\Lambda)^2$. So, the rather substantial $\kappa - 1$ needed to accommodate a_μ^{exp} would argue against a much larger Λ. Similarly, the large value of the anomalous W electric quadrupole moment $\lambda \simeq -6$ needed to reconcile $a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$ is also ruled out by collider data (which implies $|\lambda| \lesssim 0.1$). Hence, it appears that anomalous W boson properties cannot be the primary source of the discrepancy in a_μ^{exp}.

3.3.2 New Gauge Bosons

The local $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry of the Standard Model can be easily expanded to a larger gauge group with additional charged and neutral gauge bosons. Here, we consider effects due to a charged $W_R^\pm$ which couples to right-handed charged currents in generic left-right symmetric models and a neutral gauge boson, Z', which can naturally arise in higher rank GUT models such as $SO(10)$ or E_6. A general analysis of one-loop contributions to a_μ from extra gauge bosons has been carried out by Leveille [84] and the specific examples considered here were illustrated in [11]. Here, we will only discuss the likelihood of such bosons being the source of the apparent $a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$ discrepancy.

For the case of a W_R coupled to μ_R and a (very light) ν_R with gauge coupling g_R, one finds

$$a_\mu(W_R) \simeq (390 \times 10^{-11}) \frac{g_R^2}{g_2^2} \frac{m_W^2}{m_{W_R}^2}. \tag{53}$$

To accommodate the discrepancy in Eq. (24) requires $m_{W_R} \simeq m_W = 80.4$ GeV for $g_R \simeq g_2$, which is clearly ruled out by direct searches and precision measurements

which give $m_{W_R} \gtrsim 715$ GeV. Hence, $W_R^\pm$ is not a viable candidate for explaining the $a_\mu^{\rm exp}$ discrepancy.

Extra neutral gauge bosons (with diagonal $\overline{\mu}\mu$ couplings) do much worse in trying to explain $a_\mu^{\rm exp} - a_\mu^{\rm SM}$, partly because they often tend to give a contribution with opposite sign. For example, the Z_χ of $SO(10)$ leads to

$$a_\mu(Z_\chi) \simeq -6 \times 10^{-11} \left(\frac{m_Z^2}{m_{Z_\chi}^2} \right). \tag{54}$$

Given the collider constraint $m_{Z_\chi} \gtrsim 600$ GeV, that effect would be much too small to observe in $a_\mu^{\rm exp}$. Most other Z' scenarios give similar results.

An exception to the small effects from gauge bosons illustrated above is provided by non-chiral coupled bosons which connect μ and a heavy fermion F. In those cases, $\Delta a_\mu \simeq \frac{g^2}{16\pi^2}\frac{m_\mu m_F}{M^2}$, where M is the gauge boson mass. However, loop effects then give $\delta m_\mu \sim g^2 m_F$ (see the discussion in Sect. 3.2) and we have argued that in such scenarios Δa_μ should actually turn out to be $\sim m_\mu^2/M^2$. As previously pointed out in Eq. (47), $a_\mu^{\rm exp} - a_\mu^{\rm SM}$ then corresponds to $M \sim 1-2$ TeV.

Many other examples of "New Physics" contributions to a_μ have been considered in the literature. A general analysis in terms of effective interactions was presented in [85]. Specific other examples include effects due to muon compositeness [74], extra Higgs [86] bosons, leptoquarks [87, 88], bileptons [89], 2-loop pseudoscalar effects [90], compact extra dimensions [91, 92] etc. Given the apparent deviation in experiment from theory, all will certainly be revisited.

4 Outlook

After many years of experimental and theoretical toil, studies of the muon anomalous magnetic moment have entered an exciting new phase. Experiment E821 at Brookhaven has reported a 2.6 sigma difference betwen $a_\mu^{\rm exp}$ and the Standard Model prediction, $a_\mu^{\rm SM}$. That difference could be a strong hint of supersymmetry in roughly the $\tan\beta \simeq 4$, $m_{\rm SUSY} \simeq 100$ GeV $-$ $\tan\beta \simeq 40$, $m_{\rm SUSY} \simeq 450$ GeV region or perhaps an indication of radiative muon mass generation from "new physics" in the $1-2$ TeV range. Either case represents an exciting prospect with interesting implications for future experiments.

Of course, before the assertion of "new physics" can be taken seriously, the values of $a_\mu^{\rm exp}$ and $a_\mu^{\rm SM}$ should be further scrutinized and refined. In that regard, it is fortunate that ongoing analysis of existing μ^+ data should reduce the uncertainty in $a_\mu^{\rm exp}$ by

about another factor of 2.5 and similar statistical accuracy is expected from ongoing μ^- studies. In addition, ongoing analysis of $e^+e^- \to \pi^+\pi^-$ data in the ρ resonance region and future experimental studies at higher energy should significantly reduce the uncertainty in $a_\mu^{\rm SM}$ and enhance its credibility. Should a significant difference between theory and experiment persist after these improvements, it will rightfully be heralded as a harbinger of "new physics". We look forward to the anticipated confrontation.

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